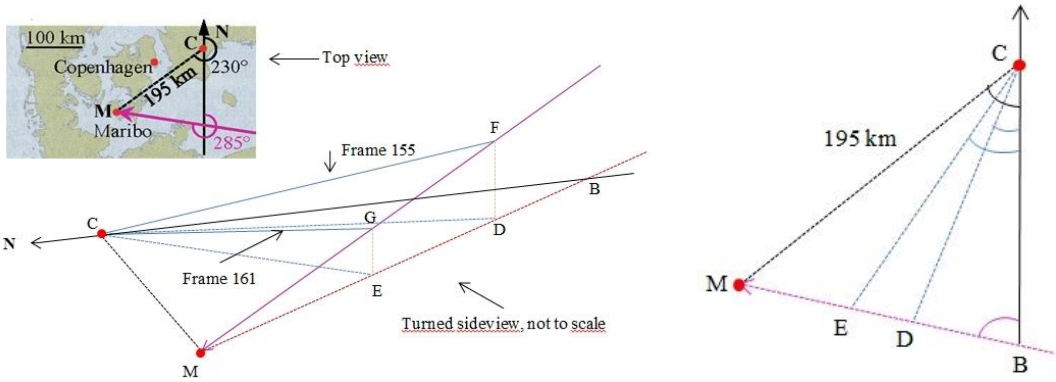
Solutions

<table>
<tr><td>1.1</td><td>


Top view: Triangle MCB: $|\mathrm{CM}| = 195$ km, $\angle\mathrm{MCB} = 230° - 180° = 50°$, and $\angle\mathrm{MBC} = 75°$, so $\angle\mathrm{CMB} = 180° - 75° - 50° = 55°$.
Then $|\mathrm{CB}| = \frac{|\mathrm{CM}|\sin(\angle\mathrm{CMB})}{\sin(\angle\mathrm{MBC})} = 165.4$ km.
Triangle DCB: $|\mathrm{CB}| = 165.4$ km, $\angle\mathrm{DCB} = 215° - 180° = 35°$,and $\angle\mathrm{DBC} = 75°$, so $\angle\mathrm{CDB} = 180° - 75° - 35° = 70°$.
Then $|\mathrm{CD}| = \frac{|\mathrm{CB}|\sin(\angle\mathrm{DBC})}{\sin(\angle\mathrm{CDB})} = 170.0$ km.
Triangle ECB: $|\mathrm{CB}| = 165.4$ km, $\angle\mathrm{ECB} = 221° - 180° = 41°$, and $\angle\mathrm{EBC} = 75°$, so $\angle\mathrm{CEB} = 180° - 75° - 41° = 64°$.
Then $|\mathrm{CE}| = \frac{|\mathrm{CB}|\sin(\angle\mathrm{EBC})}{\sin(\angle\mathrm{CEB})} = 177.7$ km.
Triangle ECD:$\angle\mathrm{ECD} = 41° - 35° = 6°$. Horisontal distance travelled by Maribo:$|\mathrm{DE}| = \frac{|\mathrm{DC}|\sin(\angle\mathrm{ECD})}{\sin(\angle\mathrm{CED})} = 19.77$ km
Side view: Triangle CFD: $|\mathrm{FD}| = |\mathrm{CD}|\tan(\angle\mathrm{FCD}) = 59.20$ km
Triangle CGE: $|\mathrm{GE}| = |\mathrm{CE}|\tan(\angle\mathrm{GCE}) = 46.62$ km
Thus vertical distance travelled by Maribo: $|\mathrm{FD}| - |\mathrm{GE}| = 12.57$ km.
Total distance travelled by Maribo from frame 155 to 161:
$|\mathrm{FG}| = \sqrt{|\mathrm{DE}|^2 + (|\mathrm{FD}| - |\mathrm{GE}|)^2} = 23.43$ km.
The speed of Maribo is $v = \frac{23.43\ \mathrm{km}}{2.28\ \mathrm{s} - 1.46\ \mathrm{s}} = 28.6\ \mathrm{km/s}$
</td><td>1.2</td></tr>
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<tr><td>1.2a</td><td>Newton's second law: $m_\mathrm{M}\frac{\mathrm{d}v}{\mathrm{d}t} = -k\rho_\mathrm{atm}\pi R_\mathrm{M}^2 v^2$ yields $\frac{1}{v^2}\mathrm{d}v = -\frac{k\rho_\mathrm{atm}\pi R_\mathrm{M}^2}{m_\mathrm{M}}\,\mathrm{d}t$.
By integration $t = \frac{m_\mathrm{M}}{k\rho_\mathrm{atm}\pi R_\mathrm{M}^2}\left(\frac{1}{0.9} - 1\right)\frac{1}{v_\mathrm{M}} = 0.88$ s.</td><td>0.7</td></tr>
<tr><td>1.2b</td><td>$\frac{E_\mathrm{kin}}{E_\mathrm{melt}} = \frac{\frac{1}{2}v_\mathrm{M}^2}{c_\mathrm{sm}(T_\mathrm{sm} - T_0) + L_\mathrm{sm}} = \frac{4.2\times10^8}{2.1\times10^6} = 2.1\times10^2 \gg 1$.</td><td>0.3</td></tr>
</table>

1.3a	$[x]=[t]^{\alpha}[\rho_{\rm sm}]^{\beta}[c_{\rm sm}]^{\gamma}\,[k_{\rm sm}]^{\delta}=[{\rm s}]^{\alpha}[{\rm kg\,m^{-3}}]^{\beta}[{\rm m^2\,s^{-2}K^{-1}}]^{\gamma}\,[{\rm kg\,m\,s^{-3}K^{-1}}]^{\delta}$, so $[{\rm m}]=[{\rm kg}]^{\beta+\delta}[{\rm m}]^{-3\beta+2\gamma+\delta}[{\rm s}]^{\alpha-2\gamma-3\delta}[{\rm K}]^{-\gamma-\delta}$. Thus $\beta+\delta=0$, $-3\beta+2\gamma+\delta=1$, $\alpha-2\gamma-3\delta=0$, and $-\gamma-\delta=0$. From which $(\alpha,\beta,\gamma,\delta)=\left(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2},+\frac{1}{2}\right)$ and $x(t)\approx\sqrt{\frac{k_{\rm sm}t}{\rho_{\rm sm}c_{\rm sm}}}$.	0.6
1.3b	$x(5\text{ s})=1.6\text{ mm}$ $\quad x/R_{\rm M}=1.6\text{ mm}/130\text{ mm}=0.012.$	0.4

1.4a	Rb-Sr decay scheme: ${}^{87}_{37}{\rm Rb}\to{}^{87}_{38}{\rm Sr}+{}^{0}_{-1}{\rm e}+\bar{\nu}_{\rm e}$	0.3
1.4b	$N_{87{\rm Rb}}(t)=N_{87{\rm Rb}}(0){\rm e}^{-\lambda t}$ and Rb→Sr: $N_{87{\rm Sr}}(t)=N_{87{\rm Sr}}(0)+[N_{87{\rm Rb}}(0)-N_{87{\rm Rb}}(t)]$. Thus $N_{87{\rm Sr}}(t)=N_{87{\rm Sr}}(0)+({\rm e}^{\lambda t}-1)N_{87{\rm Rb}}(t)$, and dividing by $N_{86{\rm Sr}}$ we obtain the equation of a straight line: $\frac{N_{87{\rm Sr}}(t)}{N_{86{\rm Sr}}}=\frac{N_{87{\rm Sr}}(0)}{N_{86{\rm Sr}}}+({\rm e}^{\lambda t}-1)\frac{N_{87{\rm Rb}}(t)}{N_{86{\rm Sr}}}.$	0.7
1.4c	Slope: ${\rm e}^{\lambda t}-1=a=\frac{0.712-0.700}{0.25}=0.050$ and $T_{1/2}=\frac{\ln(2)}{\lambda}=4.9\times10^{10}$ year. So $\tau_M=\ln(1+a)\frac{1}{\lambda}=\frac{\ln(1+a)}{\ln(2)}T_{1/2}=3.4\times10^{9}$ year .	0.4

1.5	Kepler's 3rd law on comet Encke and Earth, with the orbital semi-major axis of Encke given by $a=\frac{1}{2}(a_{\rm min}+a_{\rm max})$. Thus $t_{\rm Encke}=\left(\frac{a}{a_{\rm E}}\right)^{\frac{3}{2}}t_{\rm E}=3.30\text{ year}=1.04\times10^{8}\text{ s}.$	0.6

1.6a	For Earth around its rotation axis: Angular velocity $\omega_{\rm E}=\frac{2\pi}{24\text{ h}}=7.27\times10^{-5}\text{ s}^{-1}$. Moment of inertia $I_{\rm E}=0.83\frac{2}{5}m_{\rm E}R_{\rm E}^2=8.07\times10^{37}\text{ kg m}^2$. Angular momentum $L_{\rm E}=I_{\rm E}\omega_{\rm E}=5.87\times10^{33}\text{ kg m}^2\text{s}^{-1}$. Astroid $m_{\rm ast}=\frac{4\pi}{3}R_{\rm ast}^3\rho_{\rm ast}=1.57\times10^{15}$ kg and angular momentum $L_{\rm ast}=m_{\rm ast}v_{\rm ast}R_{\rm E}=2.51\times10^{26}\text{ kg m}^2\text{s}^{-1}$. $L_{\rm ast}$ is perpendicular to $L_{\rm E}$, so by conservation angular momentum: $\tan(\Delta\theta)=L_{\rm ast}/L_{\rm E}=4.27\times10^{-8}$. The axis tilt $\Delta\theta=4.27\times10^{-8}$ rad (so the north pole move $R_{\rm E}\,\Delta\theta=0.27$ m).	0.7
1.6b	At vertical impact $\Delta L_{\rm E}=0$ so $\Delta(I_{\rm E}\omega_{\rm E})=0$. Thus $\Delta\omega_{\rm E}=-\omega_{\rm E}(\Delta I_{\rm E})/I_{\rm E}$, and since $\Delta I_{\rm E}/I_{\rm E}=m_{\rm ast}R_{\rm E}^2/I_{\rm E}=7.9\times10^{-10}$ we obtain $\Delta\omega_{\rm E}=-5.76\times10^{-14}\text{ s}^{-1}$. The change in rotation period is $\Delta T_{\rm E}=2\pi\left(\frac{1}{\omega_{\rm E}+\Delta\omega_{\rm E}}-\frac{1}{\omega_{\rm E}}\right)\approx-2\pi\frac{\Delta\omega_{\rm E}}{\omega_{\rm E}^2}=6.84\times10^{-5}$ s.	0.7
1.6c	At tangential impact $L_{\rm ast}$ is parallel to $L_{\rm E}$ so $L_{\rm E}+L_{\rm ast}=(I_{\rm E}+\Delta I_{\rm E})(\omega_{\rm E}+\Delta\omega_{\rm E})$ and thus $\Delta T_{\rm E}=2\pi\left(\frac{1}{\omega_{\rm E}+\Delta\omega_{\rm E}}-\frac{1}{\omega_{\rm E}}\right)=2\pi\left(\frac{I_{\rm E}+\Delta I_{\rm E}}{L_{\rm E}+L_{\rm ast}}-\frac{1}{\omega_{\rm E}}\right)=-3.62\times10^{-3}$ s.	0.7

1.7a	Minimum impact speed is the escape velocity from Earth: $v_{\text{imp}}^{\text{min}} = \sqrt{\frac{2Gm_E}{R_E}} = 11.2\ \text{km/s}$	0.5
1.7b	Maximum impact speed $v_{\text{imp}}^{\text{max}}$ arises from three contributions: (I) The velocity v_{b} of the body at distance a_{E} (Earth orbit radius) from the Sun, $v_{\text{b}} = \sqrt{\frac{2Gm_S}{a_E}} = 42.1\ \text{km/s}.$ (II) The orbital velocity of the Earth, $v_{\text{E}} = \frac{2\pi a_E}{1\ \text{year}} = 29.8\ \text{km/s}.$ (III) Gravitational attraction from the Earth and kinetic energy seen from the Earth: $\frac{1}{2}(v_{\text{b}} + v_{\text{E}})^2 = -\frac{Gm_E}{R_E} + \frac{1}{2}\left(v_{\text{imp}}^{\text{max}}\right)^2.$ In conclusion: $v_{\text{imp}}^{\text{max}} = \sqrt{(v_{\text{b}} + v_{\text{E}})^2 + \frac{2Gm_E}{R_E}} = 72.8\ \text{km/s}.$	1.2

	Total	**9.0**