

### Problem 3 : Solution/marking scheme – Large Hadron Collider (10 points)

#### Part A. LHC Accelerator (6 points)

**A1 (0.7 pt)** Find the exact expression for the final velocity  $v$  of the protons as a function of the accelerating voltage  $V$ , and fundamental constants.

#### Solution A1:

[0.7]

Conservation of energy:

$$m_p \cdot c^2 + V \cdot e = m_p \cdot c^2 \cdot \gamma = \frac{m_p \cdot c^2}{\sqrt{1 - v^2/c^2}}$$

0.5

Penalties

No or incorrect total energy

-0.3

Missing rest mass

-0.2

Solve for velocity:

$$v = c \cdot \sqrt{1 - \left( \frac{m_p \cdot c^2}{m_p \cdot c^2 + V \cdot e} \right)^2}$$

0.2

without proton rest mass:

[0.5]

$$V \cdot e \simeq m_p \cdot c^2 \cdot \gamma = \frac{m_p \cdot c^2}{\sqrt{1 - v^2/c^2}}$$

0.3

Solve for velocity:

$$v = c \cdot \sqrt{1 - \left( \frac{m_p \cdot c^2}{V \cdot e} \right)^2}$$

0.2

Classical solution:

[0.2]

$$v = \sqrt{\frac{2 \cdot e \cdot V}{m_p}}$$

0.2

**A2 (0.8 pt)** For particles with high energy and low rest mass the relative deviation  $\Delta = (c - v)/c$  of the final velocity  $v$  from the speed of light is very small. Find a suitable approximation for  $\Delta$  and calculate  $\Delta$  for electrons with an energy of 60.0 GeV.

**Solution A2:**

**[0.8]**

velocity (from previous question):

$$v = c \cdot \sqrt{1 - \left( \frac{m_e \cdot c^2}{m_e \cdot c^2 + V \cdot e} \right)^2} \quad \text{or} \quad c \cdot \sqrt{1 - \left( \frac{m_e \cdot c^2}{V \cdot e} \right)^2}$$

**0.1**

relative difference:

$$\Delta = \frac{c - v}{c} = 1 - \frac{v}{c}$$

**0.1**

$$\rightarrow \Delta \simeq \frac{1}{2} \left( \frac{m_e \cdot c^2}{m_e \cdot c^2 + V \cdot e} \right)^2 \quad \text{or} \quad \frac{1}{2} \left( \frac{m_e \cdot c^2}{V \cdot e} \right)^2$$

**0.4**

relative difference

$$\Delta = 3.63 \cdot 10^{-11}$$

**0.2**

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**classical solution** gives no points

**0.0**

**A3 (1.0 pt)** Derive an expression for the uniform magnetic flux density  $B$  necessary to keep the proton beam on a circular track. The expression should only contain the energy of the protons  $E$ , the circumference  $L$ , fundamental constants and numbers. You may use suitable approximations if their effect is smaller than the precision given by the least number of significant digits. Calculate the magnetic flux density  $B$  for a proton energy of  $E = 7.00$  TeV.

**Solution A3:**

[1.0]

Balance of forces:

$$\frac{\gamma \cdot m_p \cdot v^2}{r} = \frac{m_p \cdot v^2}{r \cdot \sqrt{1 - \frac{v^2}{c^2}}} = e \cdot v \cdot B$$

0.3

In case of a mistake, partial points can be given for intermediate steps (up to max 0.2).  
Examples:

Example: Lorentz force

0.1

Example: 
$$\frac{\gamma \cdot m_p \cdot v^2}{r}$$

0.1

Energy:

$$E = (\gamma - 1) \cdot m_p \cdot c^2 \simeq \gamma \cdot m_p \cdot c^2 \rightarrow \gamma = \frac{E}{m_p c^2}$$

Therefore:

$$\frac{E \cdot v}{c^2 \cdot r} = e \cdot B$$

0.3

With

$$v \simeq c \text{ and } r = \frac{L}{2\pi}$$

follows:

$$\rightarrow B = \frac{2\pi \cdot E}{e \cdot c \cdot L}$$

0.2

Solution:

$$B = 5.50\text{T}$$

0.2

**Penalty** for  $< 2$  or  $> 4$  significant digits

-0.1

Calculation without approximations is also correct but does not give more points

$$B = \frac{2\pi \cdot m_p \cdot c}{e \cdot L} \cdot \sqrt{\left(\frac{E}{m_p \cdot c^2}\right)^2 - \left(1 + \frac{m \cdot c^2}{E}\right)^2}$$

0.5

Penalty for each algebraic mistake

-0.1

**Classical calculation** gives completely wrong result and maximum 0.3 pt

[0.3]

$$\frac{m_p \cdot v^2}{r} = e \cdot v \cdot B$$

0.1

$$B = \frac{2\pi}{L \cdot e} \sqrt{2 \cdot m_p \cdot E}$$

0.1

$$B = 0.0901\text{T}$$

0.1

**Penalty** for < 2 or > 4 significant digits

-0.1

**A4 (1.0 pt)** An accelerated charged particle radiates energy in the form of electromagnetic waves. The radiated power  $P_{rad}$  of a charged particle that circulates with a constant angular velocity depends only on its acceleration  $a$ , its charge  $q$ , the speed of light  $c$  and the permittivity of free space  $\epsilon_0$ . Use a dimensional analysis to find an expression for the radiated power  $P_{rad}$ .

**Solution A4:**

[1.0]

Ansatz:

$$P_{rad} = a^\alpha \cdot q^\beta \cdot c^\gamma \cdot \epsilon_0^\delta$$

0.2

Dimensions:  $[a]=\text{ms}^{-2}$ ,  $[q]=\text{C}=\text{As}$ ,  $[c]=\text{ms}^{-1}$ ,  $[\epsilon_0]=\text{As}(\text{Vm})^{-1}=\text{A}^2\text{s}^2(\text{Nm}^2)^{-1}=\text{A}^2\text{s}^4(\text{kgm}^3)^{-1}$

All dimensions correct

0.3

Three dimensions correct

0.2

Two dimensions correct

0.1

if dimensions: N and Coulomb  $[\epsilon_0]=\text{C}^2(\text{Nm}^2)^{-1}$

$$\frac{\text{m}^\alpha}{\text{s}^{2\alpha}} \cdot \text{C}^\beta \cdot \frac{\text{m}^\gamma}{\text{s}^\gamma} \cdot \frac{\text{C}^{2\delta}}{\text{N}^\delta \cdot \text{m}^{2\delta}} = \frac{\text{N} \cdot \text{m}}{\text{s}}$$

0.1

From this follows:

$$\text{N} : \rightarrow \delta = -1, \quad \text{C} : \rightarrow \beta + 2 \cdot \delta = 0, \quad \text{m} : \rightarrow \alpha + \gamma - 2\delta = 1, \quad \text{s} : \rightarrow 2 \cdot \alpha + \gamma = 1$$

0.2

Two equations correct

0.1

And therefore:

$$\rightarrow \alpha = 2, \beta = 2, \gamma = -3, \delta = -1$$

0.1

if dimensions: N and As  $[\epsilon_0]=\text{A}^2\text{s}^2(\text{Nm}^2)^{-1}$

$$\frac{\text{m}^\alpha}{\text{s}^{2\alpha}} \cdot \text{A}^\beta \cdot \text{s}^\beta \cdot \frac{\text{m}^\gamma}{\text{s}^\gamma} \cdot \frac{\text{A}^{2\delta} \cdot \text{s}^{2\delta}}{\text{N}^\delta \cdot \text{m}^{2\delta}} = \frac{\text{N} \cdot \text{m}}{\text{s}}$$

0.1

From this follows:

$$\text{N} : \rightarrow \delta = -1, \quad \text{A} : \rightarrow \beta + 2 \cdot \delta = 0, \quad \text{m} : \rightarrow \alpha + \gamma - 2\delta = 1, \quad \text{s} : \rightarrow -2 \cdot \alpha + \beta - \gamma + 2\delta = -1$$

0.2

Two equations correct

0.1

And therefore:

$$\rightarrow \alpha = 2, \beta = 2, \gamma = -3, \delta = -1$$

0.1

if dimensions: kg and As  $[\epsilon_0]=\text{A}^2\text{s}^4(\text{kg} \cdot \text{m}^3)^{-1}$

$$\frac{\text{m}^\alpha}{\text{s}^{2\alpha}} \cdot \text{A}^\beta \cdot \text{s}^\beta \cdot \frac{\text{m}^\gamma}{\text{s}^\gamma} \cdot \frac{\text{A}^{2\delta} \cdot \text{s}^{4\delta}}{\text{kg}^\delta \cdot \text{m}^{3\delta}} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3}$$

0.1

From this follows:

$$\text{kg} \rightarrow \delta = -1, \quad \text{A} \rightarrow \beta + 2 \cdot \delta = 0, \quad \text{m} \rightarrow \alpha + \gamma - 3\delta = 2, \quad \text{s} \rightarrow -2 \cdot \alpha + \beta - \gamma + 4\delta = -3$$

**0.2**

Two equations correct

**0.1**

And therefore:

$$\rightarrow \alpha = 2, \beta = 2, \gamma = -3, \delta = -1$$

**0.1**

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Radiated Power:

$$P_{rad} \propto \frac{a^2 \cdot q^2}{c^3 \cdot \epsilon_0}$$

**0.1**

**Other solutions with other units are possible and are accepted**

**No solution but realise that unit of charge must vanish  $\beta = 2\delta$**

**0.2**

**A5 (1.0 pt)** Calculate the total radiated power  $P_{tot}$  of the LHC for a proton energy of  $E = 7.00$  TeV (Note table 1). You may use appropriate approximations.

**Solution A5:**

[1.0]

Radiated Power:

$$P_{rad} = \frac{\gamma^4 \cdot a^2 \cdot e^2}{6\pi \cdot c^3 \cdot \epsilon_0}$$

0.1

Energy:

$$E = (\gamma - 1)m_p \cdot c^2 \quad \text{or equally valid} \quad E \simeq \gamma \cdot m_p \cdot c^2$$

0.2

Acceleration:

$$a \simeq \frac{c^2}{r} \quad \text{with} \quad r = \frac{L}{2\pi}$$

0.2

Therefore:

$$P_{rad} = \left(\frac{E}{m_p c^2} + 1\right)^4 \cdot \frac{e^2 \cdot c}{6\pi \epsilon_0 \cdot r^2} \quad \text{or} \quad \left(\frac{E}{m_p c^2}\right)^4 \cdot \frac{e^2 \cdot c}{6\pi \epsilon_0 \cdot r^2}$$

0.3

$$\text{(not required } P_{rad} = 7.94 \cdot 10^{-12} \text{W)}$$

Total radiated power:

$$P_{tot} = 2 \cdot 2808 \cdot 1.15 \cdot 10^{11} \cdot P_{rad} = 5.13 \text{kW}$$

0.2

**penalty** for missing factor 2 (for the two beams): **-0.1**

-0.1

**penalty** for wrong numbers 2808 and/or  $1.15 \cdot 10^{11}$  (numbers come from table 1): **-0.1**

-0.1

**A6 (1.5 pt)** Determine the time  $T$  that the protons need to pass through this field.

**Solution A6:**

[1.5]

2nd Newton's law

$$F = \frac{dp}{dt} \text{ leads to}$$

0.2

$$\frac{V \cdot e}{d} = \frac{p_f - p_i}{T} \text{ with } p_i = 0$$

0.3

Conservation of energy:

$$E_{tot} = m \cdot c^2 + e \cdot V$$

0.2

Since

$$E_{tot}^2 = (m \cdot c^2)^2 + (p_f \cdot c)^2$$

0.2

$$\rightarrow p_f = \frac{1}{c} \cdot \sqrt{(m \cdot c^2 + e \cdot V)^2 - (m \cdot c^2)^2} = \sqrt{2e \cdot m \cdot V + \left(\frac{e \cdot V}{c}\right)^2}$$

0.2

$$\rightarrow T = \frac{d \cdot p_f}{V \cdot e} = \frac{d}{V \cdot e} \sqrt{2e \cdot m_p \cdot V + \left(\frac{e \cdot V}{c}\right)^2}$$

0.3

$$T = 218\text{ns}$$

0.1

Alternative solution

[1.5]

2nd Newton's law

$$F = \frac{dp}{dt} \text{ leads to}$$

0.2

$$\frac{V \cdot e}{d} = \frac{p_f - p_i}{T} \text{ with } p_i = 0$$

0.3

velocity from A1 or from conservation of energy

$$v = c \cdot \sqrt{1 - \left(\frac{m_p \cdot c^2}{m_p \cdot c^2 + V \cdot e}\right)^2}$$

0.2

and hence for  $\gamma$

$$\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}} = 1 + \frac{e \cdot V}{m_p \cdot c^2}$$

0.2

$$\rightarrow p_f = \gamma \cdot m_p \cdot v = \left(1 + \frac{e \cdot V}{m_p \cdot c^2}\right) \cdot m_p \cdot c \cdot \sqrt{1 - \left(\frac{m_p \cdot c^2}{m_p \cdot c^2 + V \cdot e}\right)^2}$$

0.2

$$\rightarrow T = \frac{d \cdot p_f}{V \cdot e} = \frac{d \cdot m_p \cdot c}{V \cdot e} \cdot \sqrt{\left(\frac{m_p \cdot c^2 + e \cdot V}{m_p \cdot c^2}\right)^2 - 1} = \frac{d}{V \cdot e} \sqrt{2e \cdot m_p \cdot V + \left(\frac{e \cdot V}{c}\right)^2}$$

0.3

$$T = 218\text{ns}$$

0.1

**Alternative solution: integrate time**

[1.5]



Energy increases linearly with distance x

$$E(x) = \frac{e \cdot V \cdot x}{d} \quad 0.2$$

$$t = \int dt = \int_0^d \frac{dx}{v(x)} \quad 0.2$$

$$v(x) = c \cdot \sqrt{1 - \left( \frac{m_p \cdot c^2}{m_p \cdot c^2 + \frac{e \cdot V \cdot x}{d}} \right)^2} = c \cdot \frac{\sqrt{(m_p \cdot c^2 + \frac{e \cdot V \cdot x}{d})^2 - (m_p \cdot c^2)^2}}{m_p \cdot c^2 + \frac{e \cdot V \cdot x}{d}}$$

$$= c \cdot \frac{\sqrt{\left(1 + \frac{e \cdot V \cdot x}{d \cdot m_p \cdot c^2}\right)^2 - 1}}{1 + \frac{e \cdot V \cdot x}{d \cdot m_p \cdot c^2}} \quad 0.2$$

Substitution :  $\xi = \frac{e \cdot V \cdot x}{d \cdot m_p \cdot c^2} \quad \frac{d\xi}{dx} = \frac{e \cdot V}{d \cdot m_p \cdot c^2}$  0.2

$$\rightarrow t = \frac{1}{c} \int_0^b \frac{1 + \xi}{\sqrt{(1 + \xi)^2 - 1}} \frac{d \cdot m_p \cdot c^2}{e \cdot V} d\xi \quad b = \frac{e \cdot V}{m_p \cdot c^2} \quad 0.2$$

$$1 + \xi := \cosh(s) \quad \frac{d\xi}{ds} = \sinh(s) \quad 0.1$$

$$t = \frac{m_p \cdot c \cdot d}{e \cdot V} \int \frac{\cosh(s) \cdot \sinh(s) ds}{\sqrt{\cosh^2(s) - 1}} = \frac{m_p \cdot c \cdot d}{e \cdot V} [\sinh(s)]_{b_1}^{b_2} \quad 0.2$$

with  $b_1 = \cosh^{-1}(1), \quad b_2 = \cosh^{-1}\left(1 + \frac{e \cdot V}{m_p \cdot c^2}\right)$  0.1

$$T = 218\text{ns} \quad 0.1$$

Alternative: differential equation

[1.5]

$$F = \frac{dp}{dt} \quad 0.2$$

$$\rightarrow \frac{V \cdot e}{d} = \frac{d}{dt} \left( \frac{m \cdot v}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = \frac{m \cdot a \left(1 - \frac{v^2}{c^2}\right) + m \cdot a \frac{v^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} = \gamma^3 \cdot m \cdot a \quad 0.4$$

$$a = \ddot{s} = \frac{V \cdot e}{d \cdot m} \left(1 - \frac{\dot{s}^2}{c^2}\right)^{\frac{3}{2}} \quad 0.3$$

Ansatz :  $s(t) = \sqrt{i^2 \cdot t^2 + k} - l$  with boundary conditions  $s(0) = 0, v(0) = 0$  0.1

$$\rightarrow s(t) = \frac{c}{V \cdot e} \left( \sqrt{e^2 \cdot V^2 \cdot t^2 + c^2 \cdot m^2 \cdot d^2} - c \cdot m \cdot d \right) \quad 0.2$$

$$s = d \rightarrow T = \frac{d}{V \cdot e} \sqrt{\left(\frac{V \cdot e}{c}\right)^2 + 2V \cdot e \cdot m} \quad 0.2$$

$$T = 218\text{ns} \quad 0.1$$

classical solution:

[0.4]

$$F = \frac{V \cdot e}{d} \rightarrow \text{acceleration } a = \frac{F}{m_p} = \frac{V \cdot e}{m_p \cdot d}$$

0.1

$$d = \frac{1}{2} \cdot a \cdot T^2 \rightarrow T = \sqrt{\frac{2d}{a}}$$

0.1

And hence for the time

$$T = d \cdot \sqrt{\frac{2 \cdot m_p}{V \cdot e}}$$

0.1

$$T = 194\text{ns}$$

0.1

Part B. Particle identification (4 points)

**B1 (0.8 pt)** Express the particle rest mass  $m$  in terms of the momentum  $p$ , the flight length  $l$  and the flight time  $t$  assuming that the particles with elementary charge  $e$  travel with velocity close to  $c$  on straight tracks in the ToF detector and that it travels perpendicular to the two detection planes (see Figure 2).

**Solution B1:**

[0.8]

with velocity

$$v = \frac{l}{t}$$

0.1

relativistic momentum

$$p = \frac{m \cdot v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

0.2

gets

$$p = \frac{m \cdot l}{t \cdot \sqrt{1 - \frac{l^2}{t^2 \cdot c^2}}}$$

0.2

→ mass

$$m = \frac{p \cdot t}{l} \cdot \sqrt{1 - \frac{l^2}{t^2 \cdot c^2}} = \frac{p}{l \cdot c} \cdot \sqrt{t^2 \cdot c^2 - l^2}$$

0.3

**Alternative**

[0.8]

with flight distance:  $l$ , flight time  $t$  gets:

$$t = \frac{l}{(c \cdot \beta)}$$

0.1

relativistic momentum

$$p = \frac{m \cdot \beta \cdot c}{\sqrt{1 - \beta^2}}$$

therefore the velocity:

$$\beta = \frac{p}{\sqrt{m^2 \cdot c^2 + p^2}}$$

0.2

insert into the expression for  $t$ :

$$t = l \frac{\sqrt{m^2 \cdot c^2 + p^2}}{c \cdot p}$$

0.2

→ mass:

$$m = \sqrt{\left(\frac{p \cdot t}{l}\right)^2 - \left(\frac{p}{c}\right)^2} = \frac{p}{l \cdot c} \sqrt{(t \cdot c)^2 - l^2}$$

0.3

**non-relativistic solution:**

[0.0]

flight time:  $t = l/v$  velocity:

$$v = \frac{p}{m} \rightarrow t = \frac{l \cdot m}{p} \quad \text{and} \quad m = \frac{p \cdot t}{l}$$

this solution gives no points

0.0

**B2 (0.7 pt)** Calculate the minimal length of a ToF detector that allows to safely distinguish a charged kaon from a charged pion given both their momenta are measured to be 1.00 GeV/c. For a good separation it is required that the difference in the time-of-flight is larger than three times the time resolution of the detector. The typical resolution of a ToF detector is 150 ps (1 ps = 10<sup>-12</sup> s).

**Solution B2:**

[0.7]

Flight time difference between kaon and pion

$$\Delta t = 450\text{ps} = 450 \cdot 10^{-12}\text{s}$$

0.1

Flight time difference between kaon and pion

$$\Delta t = \frac{l}{cp}(\sqrt{m_\pi^2 \cdot c^2 + p^2} - \sqrt{m_K^2 \cdot c^2 + p^2}) = 450\text{ps} = 450 \cdot 10^{-12}\text{s}$$

0.2

$$\rightarrow l = \frac{\Delta t \cdot p}{\sqrt{m_K^2 + p^2/c^2} - \sqrt{m_\pi^2 + p^2/c^2}}$$

0.2

$$\sqrt{m_K^2 + p^2/c^2} = 1.115 \text{ GeV}/c^2 \text{ and } \sqrt{m_\pi^2 + p^2/c^2} = 1.010 \text{ GeV}/c^2$$

$$l = 450 \cdot 10^{-12} \cdot \frac{1}{1.115 - 1.010} \text{ s GeV}c^2 / (\text{GeV}c)$$

0.1

$$l = 4285.710^{-12}\text{s} \cdot c = 4285.7 \cdot 10^{-12} \cdot 2.998 \cdot 10^8\text{m} = 1.28\text{m}$$

0.1

**Penalty** for < 2 or > 4 significant digits

-0.1

**Non-relativistic solution:**

[0.3]

Flight time difference between kaon and pion

$$\Delta t = \frac{l}{p}(m_K - m_\pi) = 450\text{ps} = 450 \cdot 10^{-12}\text{s}$$

0.1

length:

$$l = \frac{\Delta t p}{m_K - m_\pi} = \frac{450 \cdot 10^{-12}\text{s} \cdot 1\text{GeV}/c}{(0.498 - 0.135)\text{GeV}/c^2}$$

0.1

$$l = 450 \cdot 10^{-12} / 0.363 \cdot c\text{s} = 450 \cdot 10^{-12} / 0.363 \cdot 2.998 \cdot 10^8\text{m}$$

$$l = 3716 \cdot 10^{-4}\text{m} = 0.372\text{m}$$

0.1

**Penalty** for < 2 or > 4 significant digits

-0.1

**B3 (1.7 pt)** Express the particle mass as a function of the magnetic flux density  $B$ , the radius  $R$  of the ToF tube, fundamental constants and the measured quantities: radius  $r$  of the track and time-of-flight  $t$ .

**Solution B3:**

[1.7]

Particle is travelling perpendicular to the beam line hence the track length is given by the length of the arc

Lorentz force  $\rightarrow$  transverse momentum, since there is no longitudinal momentum, the momentum is the same as the transverse momentum

Use formula from B1 to calculate the mass

track length: length of arc

$$l = 2 \cdot r \cdot \text{asin} \frac{R}{2 \cdot r}$$

0.5

penalty for just taking a straight track ( $l = R$ )

-0.4

partial points for intermediate steps, maximum 0.4

Lorentz force

$$\frac{\gamma \cdot m \cdot v_t^2}{r} = e \cdot v_t \cdot B \rightarrow p_T = r \cdot e \cdot B$$

0.4

partial points for intermediate steps, maximum 0.3

longitudinal momentum=0  $\rightarrow p = p_T$

0.1

momentum

$$p = e \cdot r \cdot B$$

0.1

$$m = \sqrt{\left(\frac{p \cdot t}{l}\right)^2 - \left(\frac{p}{c}\right)^2} = e \cdot r \cdot B \cdot \sqrt{\left(\frac{t}{2r \cdot \text{asin} \frac{R}{2r}}\right)^2 - \left(\frac{1}{c}\right)^2}$$

0.6

partial points for intermediate steps, maximum 0.5

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**Non-relativistic:** track length: length of arc

[0.9]

$$l = 2 \cdot r \cdot \text{asin} \frac{R}{2 \cdot r}$$

0.5

penalty for just taking a straight track ( $l = R$ )

-0.4

partial points for intermediate steps, maximum 0.4

$$m = \frac{p \cdot t}{l} = \frac{e \cdot r \cdot B \cdot t}{2r \cdot \text{asin} \frac{R}{2r}} = \frac{e \cdot B \cdot t}{2 \cdot \text{asin} \frac{R}{2r}}$$

0.4

partial points for intermediate steps, maximum 0.3

**B4 (0.8 pt)** Identify the four particles by calculating their mass.

Particle	Radius r [m]	Time of flight [ns]
A	5.10	20
B	2.94	14
C	6.06	18
D	2.32	25

**Solution B4:**

[0.8]

Particle	arc [m]	p	p	pt/l	pt/l	pt/l	Mass	Mass
		$[\frac{MeV}{c}]$	$[\frac{mkg}{s}]$ $10^{-19}$	$[\frac{MeVs}{cm}]$ $10^{-6}$	$[\frac{MeV}{c^2}]$	[kg] $10^{-27}$	$[\frac{MeV}{c^2}]$	[kg] $10^{-27}$
A	3.786	764.47	4.0855	4.038	1210.6	2.158	938.65	1.673
B	4.002	440.69	2.3552	1.542	462.2	0.824	139.32	0.248
C	3.760	908.37	4.8546	4.349	1303.7	2.324	935.10	1.667
D	4.283	347.76	1.8585	2.030	608.6	1.085	499.44	0.890

**Particles A and C are protons, B is a Pion and D a Kaon**

correct mass and identification: per particle

0.2

**penalty** for correct mass but no or wrong identification for 1 or 2 particles

-0.1

**penalty** for correct mass but no or wrong identification for 3 or 4 particles

-0.2

wrong mass, correct momentum: per particle

0.1

wrong momentum, correct arc for 3 or 4 particles

0.2

wrong momentum, correct arc for 1 or 2 particles

0.1

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**non relativistic solution  $m = pt/l$  Particle identification is not possible**

[0.4]

Particle	arc [m]	p	p	$m = p \cdot t/l$	$m = p \cdot t/l$	$m = p \cdot t/l$
		$[\frac{MeV}{c}]$	$[\frac{mkg}{s}]$ $10^{-19}$	$[\frac{MeVs}{cm}]$ $10^{-6}$	$[\frac{MeV}{c^2}]$	[kg] $10^{-27}$
A	3.786	764.47	4.0855	4.038	1210.6	2.158
B	4.010	440.69	2.3552	1.542	462.2	0.824
C	3.760	908.37	4.8546	4.349	1303.7	2.324
D	4.283	347.76	1.8585	2.030	608.6	1.085

correct mass or correct momentum: per particle

0.1

wrong momentum, correct arc for 3 or 4 particles

0.2

wrong momentum, correct arc for 1 or 2 particles

0.1